Aggregation Bias

Abstract

We propose an axiomatic approach to a decision maker's information aggregation problem. This axiomatic analysis provides a positive model of the decision maker's evidence assessment process, wherein submitted credible evidence is aggregated into an overall assessment that is responsive to the individual assessments provided. We show that the axiomatic approach produces a two-parameter family of functions and the parameters of the aggregation function have natural interpretations as the decision maker's bias against the information provider's report and the breadth of the interpretation of the decision maker's perspective. We consider two different applications and provide empirical evidence that the decision makers use this form of information aggregation in practice.

1. Introduction

Information aggregation is critical for firms due to its effect on fundamental operations and marketing decisions. For instance, in many supply chains, suppliers use demand forecast information provided by their customers together with their own estimates to determine their production capacity. However, such critical information sharing practices are usually prone to strategic manipulations like overoptimistic forecasts, which are pervasive across industries from electronics and semiconductors to medical equipment and commercial aircraft ([2], [7], [10], [11]). The cost of such inflated forecasts from customers to ensure abundant supply capacity can be significant for the manufacturers: [3] shows that inflated customer forecasts caused $2.1 billion excess inventory costs in 2001 for the major networking equipment supplier Cisco.

In response to strategic manipulation incentives or biased views of the information providers, the decision makers tend to be skeptical about the quality of shared information. For instance, in the retailing industry, category captainship is a common management practice where one of the leading manufacturers provides recommendations regarding the strategic category decisions such as pricing, promotion, shelf management, and assortment decisions. However, category captainship practices vary in terms of the extent to which the retailer implements captain's recommendations: At one end of the spectrum, some retailers use their category captains' recommendations as they are; at the other end, some retailers filter the recommendations provided by their captains and verify their appropriateness before implementing the recommendations [13]. As an another example from the automotive industry, [10] reports that General Motors "purifies" the demand forecast information received from its dealers before using it to decide on the component capacity of its assembly lines. Such conflicting incentives and interactions of supply chain firms prevent effective information sharing and, as a result, the supply chain suffers from sub-optimal operational decisions (e.g., having too much inventory or missing potential demand).

A decision maker who receives a potentially non-reliable information from an outside firm needs to consider a way to aggregate all the available information into a final assessment. The way of achieving final assessment, or simply *assessment process*, typically involves behavioral components. For instance, in many forecast information sharing arrangements, trust and trustworthiness play a key role because it is impossible to envision all contingencies and write a complete set of contracts to eliminate all possible vulnerabilities business partners face or to account for all uncertainties throughout the relationship [12]. To illustrate, consider a supply chain consisting of a manufacturer (he) and a retailer (she). The manufacturer may rely on the downstream retailer's demand forecast to secure capacity before receiving binding purchase orders from the retailer. The retailer possesses better forecast information than the manufacturer because of her proximity to the market. However, the retailer often has an incentive to inflate her forecast information to ensure abundant supply. In such a setup, the manufacturer's final assessment about the consumer demand depends on the level of trust the manufacturer puts into the retailer's forecast. The manufac-

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turer may choose to trust or ignore the retailer's forecast completely, or adjust her own judgement with the newly available forecast information. Thus, the manufacturer's final assessment decision on the market demand needs to internalize his trust behavior.

From a theoretical perspective, to mimic the behavioral aspects of information aggregation, it is common to model an uncertainty facing decision maker as a Bayesian agent; that is, the decision maker uses the information provided by its business partner to estimate the true state of the world in a very specific way. However, inasmuch as the information providers' information can be regarded as the outcome of strategic search processes in which the decision maker observes only the evidence actually presented, there is a huge amount of "missing data," which would tend to make Bayesian inference highly prior-dependent. For instance, there may be relevant evidence that is available but not presented due to the information provider's manipulation incentives (e.g., inflated forecasts from retailers to ensure abundant supply capacity). Relevant evidence may also be ruled inadmissible with respect to the decision maker's evidentiary standard (e.g., information purification by General Motors). Another aspect of missing data is the extent of the information provider's "effort" (e.g., how much time/energy/money the business partner spent on the search for evidence). Finally, the underlying true state or the world, which presumably affects the information provider's ability to find (and the cost of) exculpatory evidence, is unobservable to the decision maker. To use a Bayesian model of decision maker, one must substitute a subjective prior distribution for all of this "missing data." In addition to theoretical considerations above, laboratory studies such as [6], [9], and [11] show that participants do not use Bayes' rule to update their beliefs and the decision maker's bias on the reported information affects the aggregation outcome. Therefore, we conjecture that a decision maker who receives information from different sources follows a significantly simpler rule than Bayes' rule to generate a final assessment about decision uncertainty.

In this paper, we propose an axiomatic approach to the decision maker's information aggregation problem. The purpose of this axiomatic analysis is twofold. First, it provides a positive model of the decision maker's evidence assessment process, wherein submitted credible evidence is aggregated into an overall assessment that is responsive to the individual assessments provided. Second, the analysis produces a two-parameter family of functions that aggregates the submitted evidence into the decision maker's assessment. These parameters have natural interpretations as the decision maker's bias against the information provider's report and the breadth of the interpretation of the decision maker's perspective. Thus, we provide a non-Bayesian information aggregation methodology that is consistent with the natural restrictions of information aggregation.

To understand the impact of information aggregation process on firm decisions, we apply our aggregation methodology to two different settings. In the first setting, we consider a forecast sharing model in which a manufacturer may rely on the downstream retailer's demand forecast to secure capacity before receiving binding purchase orders from the retailer. Specifically, we characterize how the manufacturer's aggregation process affects his belief update about the private forecast information given the retailer's report. This characterization provides effective prescriptions for forecast management and contracting strategies for actual business environments where behavioral components of decision making matters. In the second setting, we empirically examine how prior product reviews affect the purchasing decisions of consumers. We focus on identifying whether consumers show any aggregation bias when they update their information about a product's quality.

2. Assessment Generation

In this section, we provide a model of decision maker's information aggregation process, wherein credible evidence from different sources is aggregated into an overall assessment. Let ξ_i and ξ_j denote the individual assessments proffered by two information sources where $\xi_i, \xi_j \in \left[\underline{\xi}, \overline{\xi}\right]$ are bounded below and above by parameters $\underline{\xi}$ and $\overline{\xi}$, respectively. For example, in the forecast sharing game analyzed in Section 3.1, the information sources are the retailer's report and the manufacturer's prior information about the demand state. Let Ξ be the assessment space, that is, $\Xi = \left[\underline{\xi}, \overline{\xi}\right] \times \left[\underline{\xi}, \overline{\xi}\right]$. Any point in Ξ represents the pair of assessments summarizing the two cases provided by the information provider i and j. The decision maker's assessment process is represented by the function $A(\xi_i, \xi_j)$ where $A : \Xi \rightarrow \left[\underline{\xi}, \overline{\xi}\right]$. In what follows we assume that A is continuous for all $(\xi_i, \xi_m) \in \Xi$, and that the indicated properties are to hold for all $(\xi_i, \xi_j) \in \Xi$.

2.1 Axioms for Aggregation

We assume that the assessment function A should embody two characteristics: (1) be responsive to the information provided and (2) reflects a notion of fairness, meaning that credible information provided by the parties should be used in an unbiased manner.

The first characteristic (*responsiveness*) is fairly straightforward to implement, which we do via the properties of *interiority* (I) and *strict monotonicity* (SM).

$$\text{(I):}\quad \max\{\xi_i,\xi_j\} \geq A(\xi_i,\xi_j) \geq \min\{\xi_i,\xi_j\}$$
$$\text{(SM):}\quad \xi_i^* > \xi_i, \xi_j^* > \xi_j \quad\Rightarrow$$
$$A(\xi_i^*,\xi_j) > A(\xi_i,\xi_j) \text{ and } A(\xi_i,\xi_j^*) > A(\xi_i,\xi_j)$$

Interiority argues that the decision maker's assessment should lie within the range of assessments provided by the information providers. To illustrate, suppose that ξ_i is greater than ξ_j. In this case, the decision maker may believe that the information provider *i*'s incentives are biased upwards and the final assessment should be less that the *i*' report. However, in this case, the decision maker has no reason to think that the true value of ξ should be less than ξ_j. Note that an implication of interiority is the property of *reflexivity*: $A(\xi,\xi) = \xi$, that is, if the *i* and *j* submits information indicating the same outcome, then the decision maker's assessment would be that same level. The second property, strict monotonicity, points out that A is strictly increasing in both ξ_i and ξ_j. That is, if any new credible information, which may be presented by the either information providers, indicates a higher ξ, then the decision maker's assessment should response to the new evidence by increasing the final assessment as well, albeit not necessarily at the same rate. Thus, strong monotonicity implies that the decision maker does not completely ignore any information provided by the sources. This condition can be interpreted as some form of decision maker's trust on the information providers reports.

The second main characteristic (*fairness*) requires that the information provided by *i* and *j* be treated in an unbiased manner. We implement this via two further axioms, *unbiasedness* (U) and *independence of presentation* (IP).

$$\text{(U):}\quad A(\lambda\xi_i,\lambda\xi_j) = \lambda A(\xi_i,\xi_j) \quad \forall\lambda, 0 < \lambda \leq 1$$
$$\text{(IP):}\quad \forall\xi_{i1},\xi_{j1},\xi_{i2},\xi_{j2}, A\left(A(\xi_{i1},\xi_{j1}), A(\xi_{i2},\xi_{j2})\right) =$$
$$A\left(A(\xi_{i1},\xi_{i2}), A(\xi_{j1},\xi_{j2})\right)$$

The unbiasedness property is a relative statement requiring that proportional scaling alone of the information should not influence the outcome disproportionately toward one party or the other. Thus, for example, if *i* and *j* both cut their estimates in half, the decision maker's assessment should fall, but there is no obvious reason why it should fall disproportionately for *i* or *j*. The seemingly natural fairness assumption is that the overall assessment should be reduced to half of the original assessment.

Finally, independence of presentation eliminates the impact of extraneous factors such as the nature, style, or sequence of presentation of the cases on the decision maker's assessment. To see this, consider the analogy that *i*'s information is partitioned (arbitrarily) into two subsets. The information from the first subset yields the assessment ξ_{i1} while the information from the remaining subset yields the assessment ξ_{i2}. Note that this does not presume $\xi_i = \xi_{i1} + \xi_{i2}$, nor does it presume $\xi_i > \xi_{i1}$ or $\xi_i > \xi_{i2}$. Similarly, let the subset assessments for *j* be denoted by ξ_{j1} and ξ_{j2}. The independence of presentation property asserts that the decision maker's assessment process should come to the same conclusion by comparison of the subsets, followed by comparisons of the assessments based on the subsets, independently of how the subsets are compared. In particular, note that on the left, ξ_{i1} is compared with ξ_{j1} and ξ_{i2} with ξ_{j2}, while on the right, ξ_{i1} and ξ_{i2} have been switched. Essentially, this axiom removes any role of procedural biases (such as how a case is presented, style of presentation, existence of technical errors) from the decision maker's assessment. We are now ready to characterize the decision maker's assessment generation function.

Theorem 1. *The family of functions, indexed by the parameters* α *and* β*, given by*

$$A(\xi_i,\xi_j;\alpha,\beta) = \begin{cases} \left(\alpha\xi_i^{\beta} + (1-\alpha)\xi_j^{\beta}\right)^{1/\beta} & \beta \neq 0, \alpha \in [0,1]; \\ \xi_i^{\alpha}\xi_j^{1-\alpha} & \beta = 0, \alpha \in [0,1]. \end{cases}$$

is the unique family of continuous functions satisfying interiority, strict monotonicity, unbiasedness, and independence of presentation.

Theorem 1 shows that any model of aggregation represented by $A(\xi_i,\xi_j;\alpha,\beta)$ that satisfies the axioms above must be a "quasi-arithmetic weighted mean." Thus, the decision maker's final assessment is a weighted average of information providers' reports where the weights are determined by the parameters α and β. A particular value of α determines the importance of *i*'s information from the decision maker's perspective. A particular value of β determines the way the decision maker averages the reports. That is, β determines what type of averaging methodology the decision maker uses. In particular, $\beta = 1$ corresponds to the weighted arithmetic mean of ξ_i and ξ_j with weights α and $(1-\alpha)$, respectively. When $\beta = 0$, the decision maker uses the weighted geometric mean and when $\beta = -1$, the final assessment is the weighted harmonic mean. Finally, $\beta \to -\infty$ yields the minimum of ξ_i and ξ_j and $\beta \to \infty$ yields the maximum of ξ_i and ξ_j.

Note that we do not restrict an information provider's report to be a single deterministic point. For instance, an information provider's report can be modeled as a random variable that is supported on some range of potential ξ values. We also allow any type of correlation or causation between the reports. For example, we do not

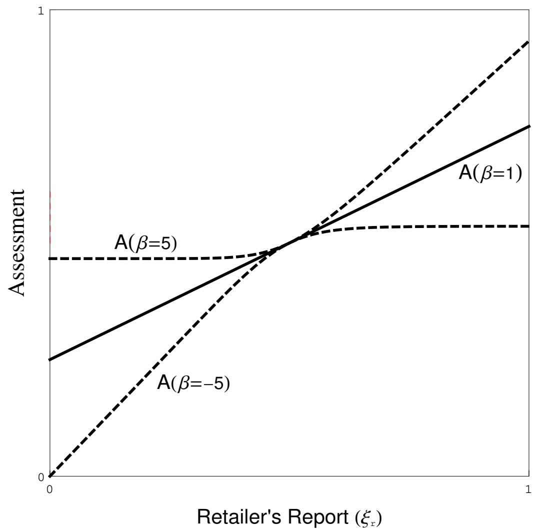


Figure 1: Change of $A(\xi_i, \xi_j; \alpha, \beta)$ in ξ_i when $\xi_j = 1/2$, $\alpha = 1/2$, and $\beta \in \{-10, 1, 10\}$.

assume that an information provider's report is independent from the other information provider's report; i.e., we allow for the cases where ξ_i is a function of ξ_j and vice versa.

2.2 Properties of Aggregation Function

Next, we characterize how the manufacturer's final assessment function reacts to the retailer's report as well as to the model parameters.

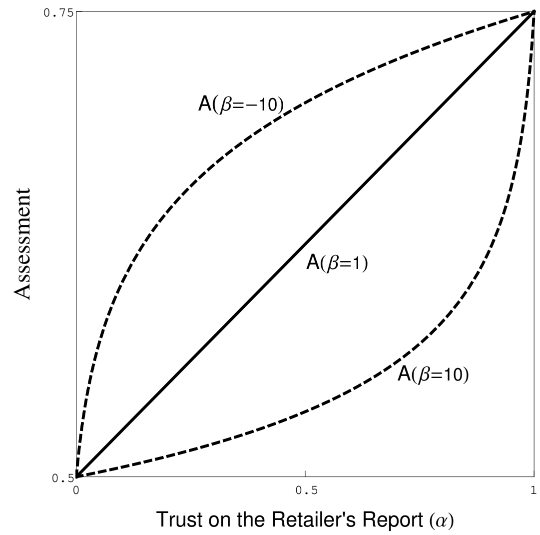
Proposition 2.

(i) $A(\xi_i, \xi_j; \alpha, \beta)$ *is a convex (concave) increasing function of* ξ_i *when* $\beta \geq 1$ ($\beta \leq 1$)*.*

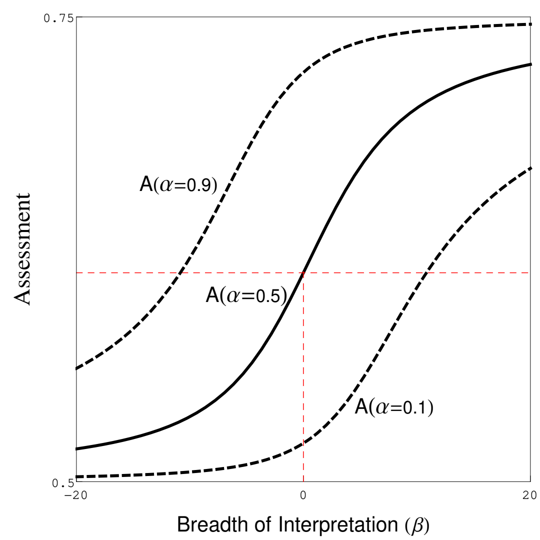
(ii) $A(\xi_i, \xi_j; \alpha, \beta)$ *is increasing in* β *for all* $\alpha \in (0, 1)$ *and* $\xi_i \neq \xi_j$*.*

(iii) $A(\xi_i, \xi_j; \alpha, \beta)$ *is increasing (decreasing) in* α *for all* $\beta \in (-\infty, \infty)$ *and* $\xi_r \geq \xi_m$ ($\xi_r < \xi_m$)*. Moreover,* $A(\xi_i, \xi_j; \alpha, \beta)$ *is convex (concave) in* α *when* $\beta \geq 1$ ($\beta \leq 1$)*.*

The first part of Proposition 2 shows that the decision maker's final assessment is an increasing function of the information providers' reports. However, the increase in the final assessment is not always linear. Depending on the value of β, the decision maker's assessment may be a convex or concave function of the reports. Figure 1 demonstrates the impact of information provider i's report on the decision maker's final assessment for different values of α and β. The solid line in Figure 1 represents the case where $\beta = 1$ so that the decision maker's final assessment is linear in ξ_i. Most information sharing models in the literature uses a setup where the final assessment is a linear function of the informed agent's report and the decision maker's own prior (e.g., models with Bayesian decision maker). However, Proposition 2 demonstrates that when $\beta > 1$ (respectively, $\beta < 1$), these models under-estimate (respectively, over-estimate) the final assessment provided by our axiomatic approach.



(a) $\xi_r = 3/4$, $\xi_m = 1/2$, and $\beta \in \{-10, 1, 10\}$



(b) $\xi_r = 3/4$, $\xi_m = 1/2$, and $\alpha \in \{0.1, 0.5, 0.9\}$

Figure 2: Change of $A(\xi_i, \xi_j; \alpha, \beta)$ in α and β.

The second part of Proposition 2 demonstrates how the decision maker's final assessment reacts to the changes in β. Intuitively, β has a natural interpretation as the breadth of the decision maker's interpretation on the reported information. A narrow interpretation means that, for any given pair (ξ_i, ξ_j), the decision maker's assessment will be lower than under a broad interpretation. Since $A(\xi_i, \xi_j; \alpha, \beta)$ is increasing in β by

Proposition 2(ii), low values of β can be interpreted as reflecting a narrow interpretation, while higher values of β reflect progressively broader interpretations. Figure 2(b) demonstrates how a change in the decision maker's interpretation impacts the final assessment for different levels of α.

A natural interpretation of α is that it represents the decision maker's relative confidence in the information providers' reports. In particular, if $\alpha = 1$, the decision maker considers that i' report is completely reliable, whereas if $\alpha = 0$, the decision maker considers that i's report is completely unreliable. Proposition 2(iii) shows that the real effect of α depends on the decision maker's interpretation; i.e., the precise value of β. Figure 2(a) demonstrates how a change in the decision maker's confidence on the i's report impacts the final assessment for different levels of interpretation. When the decision maker's interpretation is broad enough (i.e., $\beta > 1$), the final assessment is more reactive to the high values of α. On the other hand, when the decision maker's interpretation is relatively narrow (i.e., $\beta < 1$), the final assessment is more reactive to the low values of α. Only when $\beta = 1$, the final assessment is a linear function of α and the decision maker's interpretation does not play any role in the final assessment.

3. Applications

In this section, we aim to understand the impact of aggregation function derived in the previous section on two different settings in which aggregating different set of informations is required.

3.1 The Forecast Sharing Model

Consider a newsvendor model where a retailer (she) and a manufacturer (he) who interact under a wholesale price contract. The manufacturer builds capacity before demand is realized. We consider a demand model similar to [11]. In particular, demand is given by $D = \mu + \xi + \varepsilon$, where μ is a positive constant denoting the average market demand and ε is the market uncertainty. Both parties know μ, and they also know that ε is a zero-mean random variable with cumulative distribution function (c.d.f.) $F(\cdot)$ and probability density function (p.d.f.) $f(\cdot)$ supported on $[\underline{\varepsilon}, \overline{\varepsilon}]$. The parameter ξ represents the retailer's private forecast information. The retailer may have obtained this information because of her proximity to the market. The manufacturer's belief about ξ is denoted by ξ_m, which is common knowledge.

The sequence of events is as follows: (i) the retailer observes the private forecast ξ and reports her forecast information as ξ_r; (ii) by using all available information (ξ_r and ξ_m), the manufacturer generates a final assessment about ξ and builds capacity K at unit cost $c_K > 0$; (iii) demand D is realized and the retailer places an order; (iv) the manufacturer produces $\min\{D, K\}$ at unit cost $c > 0$ and charges w per unit delivered; (v) the retailer receives the order and sells at a fixed unit price $r > 0$. To ensure production is profitable, we assume $r > c + c_K$ and $w \in [c + c_K, r]$. Under this model structure, the expected profits of the retailer and manufacturer for a given K and ξ are

$$\begin{aligned} \Pi^r(K,\xi) &= (r-w)\mathbf{E}_\varepsilon[\min\{\mu+\xi+\varepsilon, K\}] \\ \Pi^m(K,\xi) &= (w-c)\mathbf{E}_\varepsilon[\min\{\mu+\xi+\varepsilon, K\}] - c_K K \end{aligned}$$

If the manufacturer knew ξ (i.e., $\xi_m = \xi$ with probability one), then he would maximize his expected profit by setting capacity as

$$K(\xi) = \mu + \xi + F^{-1}\left(\frac{w-c-c_K}{w-c}\right).$$

However, the manufacturer does not know ξ and the retailer has an incentive to distort (and possibly inflate) her report of ξ. Since the retailer's profit $\Pi^r(K,\xi)$ is increasing in the manufacturer's capacity choice K, it is in the best interest of the retailer to induce the manufacturer to build a large capacity to ensure abundant supply. Anticipating the retailer's incentive, the manufacturer would not find the reported forecast credible regardless of whether the retailer tells the truth.

We analyze this forecast sharing game under the assumption that the manufacturer uses $A(\xi_r, \xi_m; \alpha, \beta)$ to generate his assessment on the market uncertainty. In this setup, there are two decisions in sequence: the retailer's report and the manufacturer's choice of capacity. In the first stage, the retailer observes the private forecast ξ and reports her forecast information as ξ_r. In the second stage, the manufacturer updates his beliefs according his assessment process $A(\xi_r, \xi_m; \alpha, \beta)$ and builds capacity K. We analyze this forecast sharing game using backward induction and characterize first the manufacturer's optimal capacity decision followed by the retailer's optimal report strategy.

3.1.1. Analysis with Independent Reports. Suppose that the manufacturer's own judgement ξ_m is a point estimate; i.e., $\xi_m \in [\underline{\xi}, \overline{\xi}]$. Given ξ_r and ξ_m, the manufacturer decides on the capacity level to maximize his expected profit by solving

$$\max_K (w-c)\mathbf{E}_\varepsilon\left[\min\{\mu + A(\xi_r, \xi_m; \alpha, \beta) + \varepsilon, K\}\right] - c_K K$$

In this setup, the manufacturer's optimal capacity decision balances the marginal cost of adding extra capacity with the marginal expected benefit of increasing sales under the manufacturer's belief structure on the retailer's report.

Proposition 3.

(i) The manufacturer's unique optimal capacity is

$$K^* = \mu + A(\xi_r, \xi_m; \alpha, \beta) + F^{-1}\left(\frac{w-c-c_K}{w-c}\right).$$

(ii) K^* is increasing in ξ_r and decreasing in c_K.

(iii) K^* is increasing in β for all $\xi_r \neq \xi_m$ and increasing (decreasing) in α for all $\xi_r \geq \xi_m$ ($\xi_r < \xi_m$).

The first part of Proposition 3 provides the manufacturer's optimal capacity decision. The second part of Proposition 3 shows that the optimal capacity decision is positively correlated with the retailer"s report and reducing capacity cost yields higher capacity. These results are consistent with the experimental evidence provided by [11]. The last part of Proposition 3 demonstrates the impact of the manufacturer's belief structure on his capacity choice. In particular, if the manufacturer's interpretation of provided evidences about ξ becomes broader, then his optimal capacity increases. On the other hand, if his confidence on the retailer's report increases, then his optimal capacity decision moves towards to the outcome that the retailer's report suggests.

In the first stage, anticipating the manufacturer's optimal capacity strategy, the retailer maximizes her own profit by choosing a report ξ_r. As in [11], we assume that there is a cost of providing ξ_r for the retailer to capture the retailer's trustworthiness by the disutility of deception. This disutility of deception can be viewed as a psychological cost derived from the retailer's aversion to being caught in deceit. In particular, we assume that the cost of generating report ξ_r is $\kappa|\xi_r - \xi|$ when the true demand state is characterized by ξ. The parameter $\kappa > 0$ controls the retailer's incentive to misreport her private forecast. Notice that a retailer with a higher κ is more trustworthy because she incurs a higher disutility when giving the same amount of information distortion as a retailer with a lower κ. Then, the retailer solves

$$\max_{\xi_r} (r-w)\mathbf{E}_\varepsilon[\min\{\mu+\xi+\varepsilon, K^*\}] - \kappa|\xi_r - \xi|.$$

Proposition 4.

(i) The retailer's optimal reporting strategy, ξ_r^*, is such that

$$A(\xi_r^*, \xi_m; \alpha, \beta) = \xi + (z_r^* - z_m)$$

where $z_r^* = F^{-1}\left(\frac{(r-w)A'-\kappa}{(r-w)A'}\right)$, $z_m = F^{-1}\left(\frac{w-c-c_K}{w-c}\right)$, *and* $A' = \frac{\partial A(\xi_r^*, \xi_m; \alpha, \beta)}{\partial \xi_r} |_{\xi_r = \xi_r^*} = \alpha\left(\alpha + (1-\alpha)\left(\frac{\xi_m}{\xi_r^*}\right)^\beta\right)^{\frac{1-\beta}{\beta}}$.

(ii) ξ_r^* is increasing in β and it is increasing (decreasing) in α for all $\xi_r \geq \xi_m$ ($\xi_r < \xi_m$).

Part (i) of Proposition 4 reveals that the retailer's optimal reporting strategy is such that it makes the manufacturer's final assessment equal to the true demand state plus some adjustment factor. The parameter z_r^* denotes the retailer's adjustment factor, which is a function of the retailer's reporting strategy, and z_m denotes the manufacturer's adjustment factor, which is independent from the retailer's report. Then, the adjustment that the retailer makes to the true demand state is $z_r^* - z_m$. The manufacturer learns the true demand state with his assessment process (i.e., $A(\xi_r^*, \xi_m; \alpha, \beta) = \xi$) when $z_r^* = z_m$. If $z_r^* > z_m$, the manufacturer's assessment over-estimates the demand, whereas if $z_r^* < z_m$, the final assessment under-estimates the demand.

Part (ii) of Proposition 4 shows the impact of a change in the manufacturer's assessment process on the retailer's reporting strategy. In particular, the retailer inflates her report more when facing with a manufacturer whose breadth of interpretation is broader. On the other hand, if the manufacturer's confidence on the retailer increases, then it is more likely that the manufacturer's final assessment is going to be closer to the retailer's report.

3.1.2. Analysis with Dependent Reports. Our model in Section 3.1 assumes that the manufacturer's own judgement is a point estimate that is independent from the retailer's report. Now, we relax this assumption by considering a setup where the manufacturer's judgement is a function of the retailer's report; i.e., $\xi_m = h(\xi_r)$ where h is continuous function that maps each retailer's report to a belief structure. We do not put any structure on the shape of manufacturer's judgement. For instance, the manufacturer's judgement can be a point estimate that depends on the retailer' report (e.g., $h(\xi_r) = \mathbf{E}[\xi \mid \xi_r]$) or it can be a random variable where the domain of the random variable is a function of the retailer' report. The "trust-embedded" model of Ozer et al. (2011) is an example for the latter case. In their setup, the manufacturer updates his belief on ξ via the rule $\alpha\xi_r + (1-\alpha)h(\xi_r)$ where $h(\xi_r)$ follows the distribution of ξ truncated on $[\underline{\xi}, \xi_r]$.

In this section, we assume that $\beta = 1$ and the retailer knows the structure of $h(\cdot)$ for simplicity. Then, given ξ_r and $h(\cdot)$, the manufacturer's and the retailer's expected payoffs are

$$\begin{aligned}\hat{\Pi}_m &= (w-c)\mathbf{E}_\varepsilon[\min\{\mu + A(\xi_r) + \varepsilon, K\}] - c_K K \\ \hat{\Pi}_r &= (r-w)\mathbf{E}_\varepsilon[\min\{\mu + \xi + \varepsilon, K\}] - \kappa|\xi_r - \xi|\end{aligned}$$

where $A(\xi_r) = \alpha\xi_r + (1-\alpha)h(\xi_r)$. The sequence of events is the same as that in Section 3.1. In particular, there are two decisions in sequence: the retailer's report and the manufacturer's choice of capacity. As before, we analyze the model using backward induction and characterize first the manufacturer's optimal capacity decision followed by the retailer's optimal forecast report.

Proposition 5. *In the equilibrium of the model with report dependent manufacturer judgement, the manufacturer's unique optimal capacity is*

$$\hat{K}(\hat{\xi}_r^*,\alpha) = \mu + \alpha\hat{\xi}_r^* + G^{-1}\left(\frac{w-c-c_K}{w-c} \mid \hat{\xi}_r^*,\alpha\right)$$

where $G\left(\cdot \mid \hat{\xi}_r^*,\alpha\right)$ *is the c.d.f. for* $(1-\alpha)h(\hat{\xi}_r^*)+\varepsilon$ *given* $\hat{\xi}_r^*$ *and* α*, and the retailer's optimal reporting strategy,* $\hat{\xi}_r^*$*, is such that*

$$\hat{K}(\hat{\xi}_r^*,\alpha) = \xi + \hat{A}(\hat{\xi}_r^*,\alpha) + F^{-1}\left(\frac{(r-w)\hat{K}'(\hat{\xi}_r^*,\alpha)-\kappa}{(r-w)\hat{K}'(\hat{\xi}_r^*,\alpha)}\right)$$

where $\hat{K}'(\hat{\xi}_r^*,\alpha) = \frac{\partial\hat{K}(\hat{\xi}_r,\alpha)}{\partial\hat{\xi}_r}\mid_{\hat{\xi}_r=\hat{\xi}_r^*}$.

Proposition 5 generalizes the results in Section 3.1 for the cases where the manufacturer's own judgment is a function of the retailer's report. The proposition demonstrates that the incentive conflict between the manufacturer and retailer can be measured by the difference between the adjustment factors of each firm.

3.2 Customer Reviews.

The second application we consider focuses on the impact of assessment function on the customer reviews, which are increasingly important to marketing. Research has shown that ([5], [4], [8]) past reviews have impacts on future ones. In this section, we adapt the information aggregation rule to the dynamics of reviews, and analyze empirical review data to determine if the model is supported.

3.2.1. Dynamical Model with Step-wise Aggregation. Consider a setting where a particular product receives a sequence of reviews from unique individuals (i.e., no individual submits more than one review). We assume the private assessment of the product x_i for individual i is a real number and distributed i.i.d. with some cumulative distribution F_x. We further assume that an individual is influenced by the past reviews, and his "resulting" assessment is an aggregation between his private assessment, x_i, and some characteristics of the past assessments. For the purpose of this discussion, we will limit our attention to the mean of past assessments, $\bar{x}$. The formulation is general and can be extended to other characteristics, such as the most recent assessment.

The model strategy is to derive the distribution of the observed reviews from the basic assumptions of the review process, which allows us to use the maximum likelihood method to estimate the parameters of the model from empirical data. Using the aggregation rule, the resulting assessment, y, is given by:

$$y = A(x_i,\bar{x};\alpha,\beta) = \left(\alpha x_i^\beta + (1-\alpha)\bar{x}^\beta\right)^{1/\beta}$$

The cumulative distribution of y, using the change of variable $x_i = \left(\frac{y^\beta-(1-\alpha)\bar{x}^\beta}{\alpha}\right)^{1/\beta}$, is:

$$F_y(z) = F_x\left(\left(\frac{z^\beta-(1-\alpha)\bar{x}^\beta}{\alpha}\right)^{1/\beta}\right)$$

The parameter α decides how much weight an individual will place on his private assessment. If $\alpha = 1$, the individual does not use any past assessment and only rely on his/her own private assessment x_i. If $\alpha = 1$, the individual has no opinion and uses the assessment $\bar{x}$ from the past. The parameter β controls whether the response to assessments (private or past) to be concave ($\beta < 1$) or convex ($\beta > 1$). A concave/convex response can be interpreted as the individual to be more sensitive to changes in low/high assessments.

3.2.2. Reporting Bias. Past research ([1], [4], [8]) has shown that individuals do not always report their assessments of products as reviews, and that their propensity of doing so is not independent of their assessments. Specifically, the satisfaction theory ([1]) argues that an individual is more likely to report his assessment (i.e. post a review) if his/her final assessment (y) is further away from the average ($\bar{x}$). That is, the motivation to "chime in" is stronger when the individual is disagreeing with the average opinion.

We operationalize this idea with a probability of reviewing, given by:

$$P(review) = p_0 + (1-p_0)\left(1-e^{-\gamma(y-\bar{x})^2}\right)$$

Where p_0 and γ are parameters of the model. When the individual agrees with the average assessment, $y-\bar{x}=0$, and $P(review) = p_0$. In this case, p_0 can be interpreted as the minimum probability of reviewing. When the individual disagrees with the average assessment strongly, $(y-\bar{x})^2 \to \infty$, and $P(review) = 1$. That is, the individual will review with probability 1.

3.2.3. Review Data. Consumer reviews were collected from one of the most popular media website on technology and consumer electronics — CNET.com. We used a web scrawling software package BeautifulSoup in Python. Top brands in the computer category were selected: HP, Dell, Apple, Sony, Acer and Toshiba. For each brand, we collected the review scores, and the time of reviews, plus the product characteristics of all the products. This complete data set contains 18,840 reviews on 1,491 products of the 5 brands over a 7-year period. The data set is described with the statistics presented in the Table below.

Table 1: Data Description – (P.N.–Product No, R.N.–Review No., A.R.– Avg. Rating, Stdev. – Stdev. Rating)

Brands	P.N.	R.N.	A.R.	Stdev.
Apple	167	6874	3.48	1.45
Dell	193	1050	2.84	1.64
HP	390	3202	2.87	1.61
Sony	659	6930	3.60	1.47
Toshiba	82	784	3.36	1.47
Summary	1491	18840	3.23	1.51

3.2.4. Estimation and Analysis. For a particular product, we assume the private assessments of individuals are distributed with a truncated normal, with mean μ and standard deviation σ, both to be estimated from the data. We assume a truncated normal, in the range of $[0,5]$, to be consistent with the range of the review scores. We further assume the final assessment y and whether the individual reviews are conditionally independent. Since the observed review scores are in the increment of 0.5, we discretize the model, and the probability of observing a score s is given by:

$$\begin{aligned} P(s) &= P(review)P(s-1/2 < y \leq s+1/2) \\ &= P(review)(F_y(s+1/2) - F_y(s-1/2)) \end{aligned}$$

This model has 6 parameters: μ, σ, α, β, p_0 and γ. We estimate the 6 parameters by the standard maximum likelihood method. Let $\theta = \{\mu,\sigma,\alpha,\beta,p_0,\gamma\}$. For a sequence of review scores $\{s_1,s_2,...,s_N\}$, the log-likelihood function is given by:

$$L(\theta) = \sum_{i=1}^{N} log(P(s_i|\theta))$$

Note that for each term $log(P(s_i|\theta)$, $\bar{x} = \frac{1}{i-1}\sum_{j=1}^{i-1} s_j$.

3.2.5. Estimation Results and Hypothesis Testing. We only estimate the model for products with at least 50 review scores to ensure enough statistics. While the threshold is arbitrary, our main conclusions will not change if the threshold is increased to 60. The model is estimated for 72 products. It is not practical to provide the detailed estimates for all 72 products. Hence, the main consults will be supported by a summary of the estimation.

However, to ensure clarity, we first discuss the estimation of a single product: Apple iTunes 7. The following table summarizes the model estimation of this product.

Table 2: Model Estimation and Hypothesis Testing for Apple iTunes 7 – (μ–mean of private assessment, σ–s.d. of private assessment, α–weight on private assessment, β–nonlinear influence, γ–reporting bias, p_0–min probability to report)

parameter	estimate	p-value	hypothesis
μ	1.9608	NA	
σ	2.2927	NA	
α	0.8736	0.0000	H0: $\alpha = 1$
β	1.0902	0.0000	H0: $\beta = 1$
γ	0.0002	NA	
p_0	0.0008	0.0017	H0: $p_0 = 1$

Result 1: *The reviews of Apple iTune 7 exhibits nonlinear response from past influence and reporting biases.*

We use the likelihood ratio test to determine (i) if there is any influence from the past average reviews score (H0: $\alpha = 1$), (ii) if the influence from the past, if any, is nonlinear (H0: $\beta = 1$) and (iii) if there is any self-reporting bias (H0: $p_0 = 1$). We find that α is significantly below 1 (0.8736) with a p-value that is practically 0, indicating strong support that there is some influence from past reviews. β is significantly higher than 1, with a p-value also practically 0. This is strong evidence that this influence from the past is nonlinear. In addition, p_0 is significant below 1 with a p-value below 1%, again indicating strong evidence of a reporting bias, in this case.

All the products are analyzed in this fashion. Clearly, these behaviors (influence from the past, nonlinear influence, reporting bias) are not significant in all the products. We summarizes the aggregate results for the 72 products in the following table.

Result 2: *Most products exhibit influence from past reviews with half of the responses being nonlinear.*

The estimated α is significantly less than 1 for 58% of the products, indicating there is evidence of influence from past average review score in a majority of the products. Out of these products, roughly half (19 out of 42) has a significant β. It does worth noting that in all 19 cases, $\beta > 0$ consistent with the interpretation that reviewers pay more attention to extreme values.

Result 3: *Most products exhibit reporting biases.*

The estimated p_0 is significantly less than 1 for 63%

Table 3: Model Estimation and Hypothesis Testing for 72 Products

parameter	mean	s. d.	% of products significant at 5%
μ	2.948	1.277	N/A
σ	2.248	0.673	N/A
α	0.832	0.042	58%
β	1.147	0.204	26%
γ	0.807	0.811	N/A
p_0	0.057	0.087	63%

of the products, indicating reporting biases are in a majority of the products.

4. Conclusion

In this paper, we consider an axiomatic model of information aggregation. We provide a unique two-parameter family of information aggregation functions for a collection of mild axioms on the assessment generation process. The parameters of the information aggregation function have natural interpretations as certain types of consistent behavioral biases in the aggregation process. In order to test the impact of such an aggregation process on the managerial decisions, we consider two applications from the operations management and marketing literatures. In both of these applications, information aggregation is at the core of the decision making process. We show how the behavioral biases characterized by the information aggregation function affect the decision making outcomes in the context of these two applications. We also provide empirical evidence that the decision makers use this form of information aggregation in practice.

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